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Free field representations for the affine superalgebra $sl(\widehat{2/1})$ and noncritical $N = 2$ strings.

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Abstract

Free field representations of the affine superalgebra $A(1,0)^{(1)}$ at level k are needed in the description of the noncritical $N = 2$ string. The superalgebra admits two inequivalent choices of simple roots. We give the Wakimoto representations corresponding to each of these and derive the relation between the two at the quantum level.

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In recent years, noncritical strings have been the focus of intense activity. Most encouragement came from the nonperturbative definition of string theory in space-time dimension $d < 1$ in the context of matrix models. Although less powerful, the continuum approach, which involves the quantisation of the Liouville theory, gives results which are in agreement with those obtained in matrix models, on the scaling behaviour of correlators for instance [1]. A generalisation of these ideas roots are shown to be related by nonlinear canonical field transformations, both at the classical and at the quantum level. The to supersymmetric strings however is easier in the continuum. Some effort has been put in the study of $N = 1$ and $N = 2$ noncritical superstrings, but no clear picture has emerged so far as how useful they might be, in particular in extracting nonperturbative information [2, 3, 4, 5, 6]. However, the $N = 2$ noncritical string possesses interesting features and technical challenges. In particular, as emphasized in [2], the $N = 2$ noncritical string is not confined to the regime of weak gravity, i.e. the phase transition point between weak and strong gravity regimes is not of the same nature as in the $N = 0, 1$ cases. This absence of barrier in the central charge is source of complications, but also the hope of some new physics.

In this letter, we provide some of the algebraic background required to describe the space of physical states of the noncritical $N = 2$ string, from the point of view developed in [7, 8, 9, 3] for the $N = 0$ and $N = 1$ cases. It is argued there that gauged G/G Wess-Zumino-Novikov-Witten (WZNW) models, with G a Lie (super)group, are promising tools for the study of noncritical (super)strings. In particular, the $SL(2/1;\mathbf{R})/SL(2/1;\mathbf{R})$ topological quantum field theory obtained by gauging the anomaly free diagonal subgroup $SL(2/1;\mathbf{R})$ of the global $SL(2/1;\mathbf{R})_L \times SL(2/1;\mathbf{R})_R$ symmetry of the WZNW model appears to be intimately related to the noncritical charged fermionic string, which is the prototype of $N = 2$ supergravity in two dimensions. A comparison of the ghost content of the two theories strongly suggests that the $N = 2$ noncritical string is equivalent to the tensor product of a *twisted* $SL(2/1;\mathbf{R})/SL(2/1;\mathbf{R})$ WZNW model with the topological theory of a spin 1/2 system [10].

For the bosonic and fermionic noncritical strings, the gauged WZNW action is based on the Lie group $SL(2;\mathbf{R})$ and the Lie supergroup $Osp(1/2;\mathbf{R})$ respectively. It is also plausible that W_N strings are related to the $SL(N;\mathbf{R})/SL(N,\mathbf{R})$ WZNW model [11, 12]. It is however only when a one-to-one correspondence between the physical states and equivalence of the correlation functions of the two theories are established that one can view the twisted G/G model as the topological version of the corresponding noncritical string theory. For the bosonic string, the recent derivation of conformal blocks for admissible representations of $\widehat{sl(2;\mathbf{R})}$ is a major step in this direction [13].

The physical states of the $SL(2/1;\mathbf{R})/SL(2/1;\mathbf{R})$ theory will be obtained in a forthcoming publication as elements of the cohomology of the BRST charge [10]. The procedure we follow is by now quite standard [14, 7, 9, 3]. The partition function of the $SL(2/1;\mathbf{R})/SL(2/1;\mathbf{R})$ theory splits in three sectors : a level k and a level $-(k + 2)$ WZNW models based on $SL(2/1;\mathbf{R})$ as well as a system of four fermionic ghosts $(b_a, c^a), a = \pm, 3, 4$ and four bosonic ghosts $(\beta_\alpha, \gamma^\alpha), (\beta'_\alpha, \gamma'_\alpha), \alpha = \pm\frac{1}{2}$ corresponding to the four even (resp. odd) generators of $SL(2/1;\mathbf{R})$ [15, 16, 17, 7].

The cohomology is calculated on the space $\mathcal{F}_k \otimes \mathcal{F}_{-(k+2)} \otimes \mathcal{F}_2$ where $\mathcal{F}_k$ denotes the space of irreducible representations of $\hat{sl}(2/1;\mathbf{R})_k$, while $\mathcal{F}_{-(k+2)}$ and $\mathcal{F}_2$ denote the Fock spaces of the level $-(k+2)$ and ghosts sectors respectively. As a first step, one calculates the cohomology on the whole Fock space, using a free field representation of $\hat{sl}(2/1;\mathbf{R})$ and its dual. These are the Wakimoto modules presented below. Because of the non-unique interpretation of the Dynkin diagram for the Lie superalgebra $A(1, 0)$ (the complexification $sl(2/1;\mathbf{C})$ of $sl(2/1;\mathbf{R})$ in Kac's notations [18]), one can associate two different free field representations to the two Weyl inequivalent choices of simple roots. The highly non linear relations between the free fields of these two representations are worked out in detail, enabling one to obtain the physical states of the noncritical $N = 2$ string unambiguously.

In a second step, one must pass from the cohomology on the Fock space to the irreducible representations of $A(1, 0)^{(1)} \sim \hat{sl}(2/1;\mathbf{C})$ at fractional level k. Within this class of representations, only those called *admissible* [19] are of interest for the problem at end. They are irreducible, usually nonintegrable, and their

characters transform as finite representations of the modular group. A detailed analysis of the representation theory of $\hat{sl}(2/1;\mathbf{C})$ is given elsewhere [20].

The set $\mathcal{M}$ of 3×3 matrices with real entries m_{ij} whose diagonal elements satisfy the super-tracelessness condition

$$m_{11}+m_{22}-m_{33}=0 \tag{1}$$

forms, with the standard laws of matrix addition and multiplication, the real Lie superalgebra $sl(2/1;\mathbf{R})$. Any matrix $\mathbf{m}\in\mathcal{M}$ can be expressed as a real linear combination of eight basis matrices

$$\begin{aligned}\mathbf{m} &= m_{11}\mathbf{h_1}+m_{22}\mathbf{h_2}+m_{12}\mathbf{e}_{\alpha_1+\alpha_2}+m_{21}\mathbf{e}_{-(\alpha_1+\alpha_2)}\\ &+ m_{32}\mathbf{e}_{\alpha_1}+m_{23}\mathbf{e}_{-\alpha_1}+m_{13}\mathbf{e}_{\alpha_2}+m_{31}\mathbf{e}_{-\alpha_2}\end{aligned} \tag{2}$$

with

$$\begin{aligned}
\mathbf{h}_1 &= \left(\begin{array}{cc|c} 1 & 0 & 0\\ 0 & 0 & 0\\ \hline 0 & 0 & 1\end{array}\right), \qquad \mathbf{h}_2 = \left(\begin{array}{cc|c} 0 & 0 & 0\\ 0 & 1 & 0\\ \hline 0 & 0 & 1\end{array}\right),\\
\mathbf{e}_{\alpha_1+\alpha_2} &= \left(\begin{array}{cc|c} 0 & 1 & 0\\ 0 & 0 & 0\\ \hline 0 & 0 & 0\end{array}\right), \qquad \mathbf{e}_{-(\alpha_1+\alpha_2)} = \left(\begin{array}{cc|c} 0 & 0 & 0\\ 1 & 0 & 0\\ \hline 0 & 0 & 0\end{array}\right),\\
\mathbf{e}_{\alpha_1} &= \left(\begin{array}{cc|c} 0 & 0 & 0\\ 0 & 0 & 0\\ \hline 0 & 1 & 0\end{array}\right), \qquad \mathbf{e}_{-\alpha_1} = \left(\begin{array}{cc|c} 0 & 0 & 0\\ 0 & 0 & 1\\ \hline 0 & 0 & 0\end{array}\right),\\
\mathbf{e}_{\alpha_2} &= \left(\begin{array}{cc|c} 0 & 0 & 1\\ 0 & 0 & 0\\ \hline 0 & 0 & 0\end{array}\right), \qquad \mathbf{e}_{-\alpha_2} = \left(\begin{array}{cc|c} 0 & 0 & 0\\ 0 & 0 & 0\\ \hline 1 & 0 & 0\end{array}\right).
\end{aligned} \tag{3}$$

From this fundamental 3-dimensional representation of $sl(2/1;\mathbf{R})$, one can write down the (anti)-commutation relations obeyed by its four bosonic generators $H_\pm, E_{\pm(\alpha_1+\alpha_2)}$ (corresponding to the even basis matrices $\mathbf{h}_\pm=\mathbf{h_1}\pm\mathbf{h_2}, \mathbf{e}_{\pm(\alpha_1+\alpha_2)}$) and its four fermionic generators $E_{\pm\alpha_1}, E_{\pm\alpha_2}$ (corresponding to the odd basis matrices),

$$\begin{aligned}
&[E_{\alpha_1+\alpha_2}, E_{-(\alpha_1+\alpha_2)}] = H_1-H_2 \quad , \quad [H_1-H_2, E_{\pm(\alpha_1+\alpha_2)}] = \pm 2E_{\pm(\alpha_1+\alpha_2)},\\
&[E_{\pm(\alpha_1+\alpha_2)}, E_{\mp\alpha_1}] = \pm E_{\pm\alpha_2} \quad , \quad [E_{\pm(\alpha_1+\alpha_2)}, E_{\mp\alpha_2}] = \mp E_{\pm\alpha_1},\\
&[H_1-H_2, E_{\pm\alpha_1}] = \pm E_{\pm\alpha_1} \quad , \quad [H_1-H_2, E_{\pm\alpha_2}] = \pm E_{\pm\alpha_2},\\
&[H_1+H_2, E_{\pm\alpha_1}] = \pm E_{\pm\alpha_1} \quad , \quad [H_1+H_2, E_{\pm\alpha_2}] = \mp E_{\pm\alpha_2},\\
&\{E_{\alpha_1}, E_{-\alpha_1}\} = H_2 \quad , \quad \{E_{\alpha_2}, E_{-\alpha_2}\} = H_1 \quad , \quad \{E_{\pm\alpha_1}, E_{\pm\alpha_2}\} = E_{\pm(\alpha_1+\alpha_2)}.
\end{aligned} \tag{4}$$

The fermionic nonzero roots $\pm\alpha_1, \pm\alpha_2$ have length square zero, and we normalise the bosonic nonzero roots $\pm(\alpha_1+\alpha_2)$ to have length square 2. The root diagram can be represented in a 2-dimensional Minkowski space with the fermionic roots in the lightlike directions, as in Fig.1.

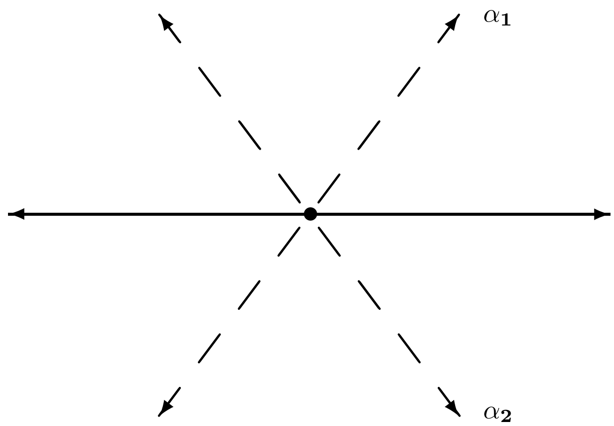

Fig.1: *The root diagram of* $A(1,0)$

The Weyl group of $sl(2/1;\mathbf{R})$ is isomorphic to the Weyl group of its even simple subalgebra $sl(2;\mathbf{R})$. There is no obvious concept of a Weyl reflection about the hyperplane orthogonal to a zero square norm fermionic root. If one therefore chooses a purely fermionic system of simple roots $\{\alpha_1, \alpha_2\}$, there is no element of the Weyl group which can transform it into the system of simple roots $\{-\alpha_2, \alpha_1+\alpha_2\}$. Dobrev and Petkova [21] and later, Penkov and Serganova [22] have actually extended the definition of the Weyl group to incorporate the transformation $\alpha_2 \to -\alpha_2$. This non uniqueness of the generalized Dynkin diagram for Lie superalgebras is well established [23]. Let us now determine the explicit relation between the Wakimoto representations of the affine version of $sl(2/1;\mathbf{R})$ constructed with both choices of simple roots.

A standard way to construct a Wakimoto free field representation of the classical Poisson bracket $sl(2/1;\mathbf{R})$ algebra is to start with a Wess-Zumino-Witten-Novikov (WZWN) model based on the non-compact simple Lie supergroup $SL(2/1;\mathbf{R})$, introduce a Gauss decomposition for the supergroup elements, and calculate the currents associated with the Kac-Moody symmetries of the WZWN action [24, 25]. Because of the non unique choice (up to Weyl transformations) of the simple roots in $sl(2/1;\mathbf{R})$, any supergroup element g can be Gauss decomposed in different ways, which lead to different free field representations. Our aim is to clarify the relation between such different representations, both at the classical and at the quantum level. Let us first summarise the results of [25].

The WZWN action for the Lie supergroup $SL(2/1;\mathbf{R})$ in the light cone gauge is given by

$$S(g) = \frac{\kappa}{2}\int dx_+dx_-\mathrm{Str}(g^{-1}\partial_+ g g^{-1}\partial_- g) + \kappa\int dx_3dx_+dx_-\mathrm{Str}(g^{-1}\partial_3 g[g^{-1}\partial_+ g, g^{-1}\partial_- g]) \qquad (5)$$

where the supertrace is the invariant, non-degenerate bilinear form on $sl(2/1;\mathbf{R})$, and κ is a parameter related to the level k of the Lie superalgebra by the relation $k=-4\pi\kappa$. The field $g(x_+,x_-)$ takes values in the connected real Lie supergroup $SL(2/1;\mathbf{R})$ and can be parametrised in a neighbourhood of the identity as $g=ABC$ with

$$\begin{aligned}
A &= \exp[\psi_1'\mathbf{e}_{-\alpha_1} + \psi_2'\mathbf{e}_{-\alpha_2} + \gamma\mathbf{e}_{-(\alpha_1+\alpha_2)}] \\
B &= \exp[\frac{1}{\sqrt{2k}}\phi_-\mathbf{h}_- + \frac{1}{\sqrt{2k}}\phi_+\mathbf{h}_+] \\
C &= \exp[\psi_1\mathbf{e}_{\alpha_1} + \psi_2\mathbf{e}_{\alpha_2} + f\mathbf{e}_{\alpha_1+\alpha_2}],
\end{aligned} \qquad (6)$$

or as $g=A'B'C'$ with

$$A' = \exp[\tilde{\psi}_1'\mathbf{e}_{-\alpha_1} + \tilde{\psi}_2'\mathbf{e}_{\alpha_2} + \tilde{\gamma}\mathbf{e}_{-(\alpha_1+\alpha_2)}]$$

$$\begin{aligned} B' &= \exp[\frac{1}{\sqrt{2k}}\tilde{\phi}_-\mathbf{h}_- + \frac{1}{\sqrt{2k}}\tilde{\phi}_+\mathbf{h}_+] \\ C' &= \exp[\tilde{\psi}_1\mathbf{e}_{\alpha_1} + \tilde{\psi}_2\mathbf{e}_{-\alpha_2} + \tilde{f}\mathbf{e}_{\alpha_1+\alpha_2}], \end{aligned} \tag{7}$$

with $\tilde{\psi}'_{1,2}, \tilde{\psi}_{1,2}, \psi'_{1,2}$ and $\psi_{1,2}$ fermionic parametrisation fields, and $\tilde{\phi}_{1,2}, \tilde{\gamma}, \tilde{f}, \phi_{1,2}, \gamma, f$ bosonic parametrisation fields in the variables $x_\pm$. These two parametrisations, which will be referred to as type I and type II Gauss decompositions of g, single out two inequivalent ways of choosing the nilpotent (Borel) subalgebras of lowering (factors A and A') and raising (factors C and C') operators in $sl(2/1;\mathbf{R})$ according to the choice of simple roots. For instance, for a purely fermionic choice of simple roots $\{\alpha_1, \alpha_2\}$, the negative roots are $\{-\alpha_1, -\alpha_2, -(\alpha_1+\alpha_2)\}$ and the Borel subalgebra of lowering operators is therefore $\{E_{-\alpha_1}, E_{-\alpha_2}, E_{-(\alpha_1+\alpha_2)}\}$. For the non Weyl-equivalent choice $\{-\alpha_2, \alpha_1+\alpha_2\}$, the negative roots are $\{-\alpha_1, \alpha_2, -(\alpha_1+\alpha_2)\}$ and the corresponding Borel subalgebra is $\{E_{-\alpha_1}, E_{\alpha_2}, E_{-(\alpha_1+\alpha_2)}\}$

In order to relate the parametrisation fields in the type I and type II decompositions, it suffices to exploit the equality $ABC = A'B'C'$. A tedious but straightforward calculation, using the Hausdorff-Campbell formula as well as the identity

$$\exp[\tilde{\psi}'_2\mathbf{e}_{\alpha_2}]\exp[\tilde{\phi}\cdot\mathbf{h}]\exp[\tilde{\psi}_2\mathbf{e}_{-\alpha_2}] = \exp[\alpha\mathbf{e}_{-\alpha_2}]\exp[\tilde{\phi}\cdot\mathbf{h}]\exp[\frac{1}{2}\alpha'\tilde{\psi}_2(\mathbf{h}_- + \mathbf{h}_+)]\exp[\alpha'\mathbf{e}_{\alpha_2}] \tag{8}$$

with

$$\begin{aligned} \tilde{\phi}\cdot\mathbf{h} &= \frac{1}{\sqrt{2k}}(\tilde{\phi}_-\mathbf{h}_- + \tilde{\phi}_+\mathbf{h}_+) \\ \alpha = \exp[-\frac{1}{\sqrt{2k}}(\tilde{\phi}_- - \tilde{\phi}_+)]\tilde{\psi}_2 \quad &, \quad \alpha' = \exp[-\frac{1}{\sqrt{2k}}(\tilde{\phi}_- - \tilde{\phi}_+)]\tilde{\psi}'_2, \end{aligned} \tag{9}$$

provides the following classical relations,

$$\begin{aligned} f = \tilde{f} - \frac{1}{2}(\tilde{\psi}_1 - \frac{1}{2}\tilde{f}\tilde{\psi}_2)e^{-\frac{1}{\sqrt{2k}}(\tilde{\phi}_- - \tilde{\phi}_+)}\tilde{\psi}'_2 \quad &, \quad \gamma = \tilde{\gamma} + \frac{1}{2}(\tilde{\psi}'_1 - \frac{1}{2}\tilde{\gamma}\tilde{\psi}'_2)e^{-\frac{1}{\sqrt{2k}}(\tilde{\phi}_- - \tilde{\phi}_+)}\tilde{\psi}_2 \\ \phi_- = \tilde{\phi}_- + \sqrt{\frac{k}{2}}e^{-\frac{1}{\sqrt{2k}}(\tilde{\phi}_- - \tilde{\phi}_+)}\tilde{\psi}'_2\tilde{\psi}_2 \quad &, \quad \phi_+ = \tilde{\phi}_+ + \sqrt{\frac{k}{2}}e^{-\frac{1}{\sqrt{2k}}(\tilde{\phi}_- - \tilde{\phi}_+)}\tilde{\psi}'_2\tilde{\psi}_2 \\ \psi'_1 = \tilde{\psi}'_1 - \frac{1}{2}\tilde{\gamma}\tilde{\psi}'_2 \quad &, \quad \psi_1 = \tilde{\psi}_1 - \frac{1}{2}\tilde{f}\tilde{\psi}_2 \\ \psi'_2 = e^{-\frac{1}{\sqrt{2k}}(\tilde{\phi}_- - \tilde{\phi}_+)}\tilde{\psi}_2 \quad &, \quad \psi_2 = e^{-\frac{1}{\sqrt{2k}}(\tilde{\phi}_- - \tilde{\phi}_+)}\tilde{\psi}'_1, \end{aligned} \tag{10}$$

which are the key to understand the link between the Wakimoto representations associated to the type I and II decompositions, as we shall now describe.

Let us first stress that the relations (10) provide canonical transformations for the Poisson brackets of the parametrisation fields and their conjugate momenta. Indeed, when use is made of the Gauss decomposition in the WZWN action, the Polyakov-Wiegmann identity for $S(ABC)$ [15] reduces to

$$S(ABC) = S(B) + \kappa\int dx_+dx_-\mathrm{Str}(A^{-1}(\partial_-A)B(\partial_+C)C^{-1}B^{-1}). \tag{11}$$

$S(B)$ is the free field action, which can be calculated in terms of the parametrisation fields ϕ_-, ϕ_+ using (5) and (6):

$$S(B) = -\frac{1}{8\pi}\int dx_+dx_-[\partial_+\phi_-\partial_-\phi_- - \partial_+\phi_+\partial_-\phi_+], \tag{12}$$

while the second term in (11) gives the interaction as

$$\kappa \int dx_+ dx_- \quad \{ \quad e^{\sqrt{\frac{2}{k}}\phi_-}[\partial_-\gamma + \frac{1}{2}(\partial_-\psi_2')\psi_1' - \frac{1}{2}\psi_2'\partial_-\psi_1'] \, [\partial_+ f - \frac{1}{2}(\partial_+\psi_2)\psi_1 + \frac{1}{2}\psi_2\partial_+\psi_1]$$
$$+ e^{\frac{1}{\sqrt{2k}}(\phi_- + \phi_+)}\partial_-\psi_1'\partial_+\psi_1 - e^{\frac{1}{\sqrt{2k}}(\phi_- - \phi_+)}\partial_-\psi_2'\partial_+\psi_2\}. \tag{13}$$

The classical momenta conjugated to the various parametrisation fields can be easily derived from the action $S(ABC)$. One has,

$$\begin{aligned}
\Pi_{\phi_-} &= 4\pi\frac{\partial S}{\partial(\partial_-\phi_-)} = -\frac{1}{2}\partial_+\phi_- \quad , \quad \Pi_{\phi_+} = 4\pi\frac{\partial S}{\partial(\partial_-\phi_+)} = \frac{1}{2}\partial_+\phi_+ \\
\Pi_\gamma &= 4\pi\frac{\partial S}{\partial(\partial_-\gamma)} = -ke^{\sqrt{\frac{2}{k}}\phi_-}(\partial_+ f + \frac{1}{2}(\psi_1\partial_+\psi_2 - (\partial_+\psi_1)\psi_2) \equiv \beta \\
\Pi_{\psi_1'} &= 4\pi\frac{\partial S}{\partial(\partial_-\psi_1')} = \frac{\beta}{2}\psi_2' - ke^{\frac{1}{\sqrt{2k}}(\phi_- + \phi_+)}\partial_+\psi_1 \equiv \psi_1^{'\dagger} \\
\Pi_{\psi_2'} &= 4\pi\frac{\partial S}{\partial(\partial_-\psi_2')} = \frac{\beta}{2}\psi_1' + ke^{-\frac{1}{\sqrt{2k}}(\phi_- - \phi_+)}\partial_+\psi_2 \equiv \psi_2^{'\dagger}.
\end{aligned} \tag{14}$$

If one instead starts with $S(A'B'C')$, a very similar procedure leads to the following conjugate momenta,

$$\begin{aligned}
\Pi_{\tilde\phi_-} &= 4\pi\frac{\partial S}{\partial(\partial_-\tilde\phi_-)} = -\frac{1}{2}\partial_+\tilde\phi_- \quad , \quad \Pi_{\tilde\phi_+} = 4\pi\frac{\partial S}{\partial(\partial_-\tilde\phi_+)} = \frac{1}{2}\partial_+\tilde\phi_+ \\
\Pi_{\tilde\gamma} &= 4\pi\frac{\partial S}{\partial(\partial_-\tilde\gamma)} = -ke^{\sqrt{\frac{2}{k}}\tilde\phi_-}\partial_+\tilde f - \frac{1}{2}\tilde\psi_1^{'\dagger}\tilde\psi_2' \equiv \tilde\beta \\
\Pi_{\tilde\psi_1'} &= 4\pi\frac{\partial S}{\partial(\partial_-\tilde\psi_1')} = -ke^{\frac{1}{\sqrt{2k}}(\tilde\phi_- + \tilde\phi_+)}(-\frac{1}{2}\tilde f\partial_+\tilde\psi_2 + \frac{1}{2}\tilde\psi_2\partial_+\tilde f + \partial_+\tilde\psi_1) \equiv \tilde\psi_1^{'\dagger} \\
\Pi_{\tilde\psi_2'} &= 4\pi\frac{\partial S}{\partial(\partial_-\tilde\psi_2')} = -ke^{-\frac{1}{\sqrt{2k}}(\tilde\phi_- - \tilde\phi_+)}\partial_+\tilde\psi_2 - \frac{1}{2}\tilde\gamma\tilde\psi_1^{'\dagger} \equiv \tilde\psi_2^{'\dagger},
\end{aligned} \tag{15}$$

and it is easy to obtain the following relations between the type I and II conjugate momenta from (10),

$$\begin{aligned}
\beta &= \tilde\beta - \frac{1}{2}\tilde\psi_1^{'\dagger}\tilde\psi_2' \\
\psi_1^{'\dagger} &= \tilde\psi_1^{'\dagger} - \frac{1}{2}\tilde\beta e^{-\frac{1}{\sqrt{2k}}(\tilde\phi_- - \tilde\phi_+)}\tilde\psi_2 + \frac{1}{4}e^{-\frac{1}{\sqrt{2k}}(\tilde\phi_- - \tilde\phi_+)}\tilde\psi_1^{'\dagger}\tilde\psi_2'\tilde\psi_2 \\
\psi_2^{'\dagger} &= k\partial_+\tilde\psi_2' - \sqrt{\frac{k}{2}}\tilde\psi_2'(\partial_+\tilde\phi_- - \partial_+\tilde\phi_+) + \frac{1}{2}\tilde\beta\tilde\psi_1' - \frac{1}{4}\tilde\beta\tilde\gamma\tilde\psi_2' - \frac{1}{4}\tilde\psi_1^{'\dagger}\tilde\psi_2'\tilde\psi_1'
\end{aligned} \tag{16}$$

The fundamental Poisson brackets for the two Gauss decompositions are taken to be,

$$\begin{aligned}
\{\psi_i'(x), \psi_j^{'\dagger}(y)\}_{P.B.} &= -\delta_{ij}\delta(x-y) = \{\tilde\psi_i'(x), \tilde\psi_j^{'\dagger}(y)\}_{P.B.} \\
\{\gamma(x), \beta(y)\}_{P.B.} &= \delta(x-y) = \{\tilde\gamma(x), \tilde\beta(y)\}_{P.B.} \\
\{\partial_+\phi_a(x), \partial_+\phi_b(y)\}_{P.B.} &= \eta_{ab}\delta'(x-y) = \{\partial_+\tilde\phi_a(x), \partial_+\tilde\phi_b(y)\}_{P.B.}
\end{aligned} \tag{17}$$

with $\eta_{ab} = \mathrm{diag}(-1,1), a, b = -, +$, and they are related by the transformations (10) and (16).

The currents associated with the Kac-Moody symmetries of the WZWN action provide a free field representation of the classical Poisson bracket algebra $sl(2/1;\mathbf{R})$. Indeed, using the Noether method, they can be constructed from the action (5) as,

$$J(\boldsymbol{\lambda}) = -k\mathrm{Str}(\boldsymbol{\lambda}\partial_+ g g^{-1}) \tag{18}$$

where $\boldsymbol{\lambda}$ runs over the set of basis matrices (3). For the type I decomposition, the free field representation is given by ($\partial_+ \equiv \partial$),

$$
\begin{aligned}
J(\mathbf{h}_+) &= \sqrt{2k}\partial\phi_+ + \psi_1^{'\dagger}\psi_1' - \psi_2^{'\dagger}\psi_2' \\
J(\mathbf{h}_-) &= -\sqrt{2k}\partial\phi_- + \psi_1^{'\dagger}\psi_1' + \psi_2^{'\dagger}\psi_2' - 2\beta\gamma \\
J(\mathbf{e}_{\alpha_1+\alpha_2}) &= -\sqrt{2k}\gamma\partial\phi_- + \sqrt{\frac{k}{2}}\psi_1'\psi_2'\partial\phi_+ + \frac{k}{2}(\partial\psi_1'\psi_2' + \partial\psi_2'\psi_1') \\
&\quad -k\partial\gamma + \gamma(\psi_1^{'\dagger}\psi_1' + \psi_2^{'\dagger}\psi_2') - \beta\gamma^2 \\
J(\mathbf{e}_{-(\alpha_1+\alpha_2)}) &= \beta \\
J(\mathbf{e}_{\alpha_1}) &= -\sqrt{\frac{k}{2}}\psi_1'(\partial\phi_- + \partial\phi_+) - k\partial\psi_1' - \gamma\psi_2^{'\dagger} + \frac{1}{2}\psi_2^{'\dagger}\psi_2'\psi_1' - \frac{1}{2}\beta\gamma\psi_1' \\
J(\mathbf{e}_{-\alpha_1}) &= \psi_1^{'\dagger} + \frac{1}{2}\beta\psi_2' \\
J(\mathbf{e}_{\alpha_2}) &= -\sqrt{\frac{k}{2}}\psi_2'(\partial\phi_- - \partial\phi_+) - k\partial\psi_2' - \gamma\psi_1^{'\dagger} + \frac{1}{2}\psi_1^{'\dagger}\psi_1'\psi_2' - \frac{1}{2}\beta\gamma\psi_2' \\
J(\mathbf{e}_{-\alpha_2}) &= -\psi_2^{'\dagger} - \frac{1}{2}\beta\psi_1'.
\end{aligned}
\tag{19}
$$

For type II, one gets,

$$
\begin{aligned}
J(\mathbf{h}_+) &= \sqrt{2k}\partial\tilde{\phi}_+ + \tilde{\psi}_1^{'\dagger}\tilde{\psi}_1' + \tilde{\psi}_2^{'\dagger}\tilde{\psi}_2' \\
J(\mathbf{h}_-) &= -\sqrt{2k}\partial\tilde{\phi}_- + \tilde{\psi}_1^{'\dagger}\tilde{\psi}_1' - \tilde{\psi}_2^{'\dagger}\tilde{\psi}_2' - 2\tilde{\beta}\tilde{\gamma} \\
J(\mathbf{e}_{\alpha_1+\alpha_2}) &= -\sqrt{2k}\tilde{\gamma}\partial\tilde{\phi}_- - k\partial\tilde{\gamma} + \frac{1}{2}\tilde{\gamma}(\tilde{\psi}_1^{'\dagger}\tilde{\psi}_1' - \tilde{\psi}_2^{'\dagger}\tilde{\psi}_2') - \tilde{\beta}\tilde{\gamma}^2 + \tilde{\psi}_1'\tilde{\psi}_1^{'\dagger} - \frac{1}{4}\tilde{\gamma}^2\tilde{\psi}_1^{'\dagger}\tilde{\psi}_2' \\
J(\mathbf{e}_{-(\alpha_1+\alpha_2)}) &= \tilde{\beta} - \frac{1}{2}\tilde{\psi}_1^{'\dagger}\tilde{\psi}_2' \\
J(\mathbf{e}_{\alpha_1}) &= -\sqrt{\frac{k}{2}}\tilde{\psi}_1'(\partial\tilde{\phi}_- + \partial\tilde{\phi}_+) + \frac{1}{2}\sqrt{\frac{k}{2}}\tilde{\gamma}\tilde{\psi}_2'(3\partial\tilde{\phi}_- - \partial\tilde{\phi}_+) - \frac{k}{2}(\tilde{\gamma}\partial\tilde{\psi}_2' - \partial\tilde{\gamma}\tilde{\psi}_2') \\
&\quad -k\partial\tilde{\psi}_1' + \frac{1}{2}\tilde{\beta}\tilde{\gamma}^2\tilde{\psi}_2' - \tilde{\beta}\tilde{\gamma}\tilde{\psi}_1' + \tilde{\psi}_2^{'\dagger}\tilde{\psi}_1'\tilde{\psi}_2' \\
J(\mathbf{e}_{-\alpha_1}) &= \tilde{\psi}_1^{'\dagger} \\
J(\mathbf{e}_{\alpha_2}) &= \tilde{\psi}_2^{'\dagger} - \frac{1}{2}\tilde{\gamma}\tilde{\psi}_1^{'\dagger} \\
J(\mathbf{e}_{-\alpha_2}) &= \sqrt{\frac{k}{2}}\tilde{\psi}_2'(\partial\tilde{\phi}_- - \partial\tilde{\phi}_+) - k\partial\tilde{\psi}_2' - \tilde{\beta}\tilde{\psi}_1^{'\dagger} + \frac{1}{2}\tilde{\psi}_1^{'\dagger}\tilde{\psi}_2'\tilde{\psi}_1' + \frac{1}{2}\tilde{\beta}\tilde{\gamma}\tilde{\psi}_2'.
\end{aligned}
\tag{20}
$$

These two free field representations are related through (10) and (16), as first discussed in [25].

At the quantum level, the free fields become operators whose short distance behaviour is governed by the following expressions,

$$
\begin{aligned}
\psi_i'(z)\psi_j^{'\dagger}(w) &\sim -\frac{\delta_{ij}}{z-w} \sim \tilde{\psi}_i'(z)\tilde{\psi}_j^{'\dagger}(w) \\
\gamma(z)\beta(w) &\sim \frac{1}{z-w} \sim \tilde{\gamma}(z)\tilde{\beta}(w) \\
\partial\phi_a(z)\partial\phi_b(w) &\sim \frac{\eta_{ab}}{(z-w)^2} \sim \partial\tilde{\phi}_a(z)\partial\tilde{\phi}_b(w).
\end{aligned}
\tag{21}
$$

The "quantum" momenta are very similar to the classical conjugate momenta,

$$\begin{aligned}
\beta &= -(k+\frac{1}{2})e^{-2\alpha_-\phi_-}(\partial_+ f + \frac{1}{2}(\psi_1\partial_+\psi_2 - (\partial_+\psi_1)\psi_2) \\
{\psi_1'}^\dagger &= \frac{\beta}{2}\psi_2' - (k+\frac{1}{2})e^{-\alpha_-(\phi_-+\phi_+)}\partial_+\psi_1 \\
{\psi_2'}^\dagger &= \frac{\beta}{2}\psi_1' + (k+\frac{1}{2})e^{\alpha_-(\phi_--\phi_+)}\partial_+\psi_2 \\
\tilde\beta &= -(k+\frac{1}{2})e^{-2\alpha_-\tilde\phi_-}\partial_+\tilde f - \frac{1}{2}\tilde{\psi}_1'^\dagger\tilde\psi_2' \\
\tilde{\psi}_1'^\dagger &= -(k+\frac{1}{2})e^{-\alpha_-(\tilde\phi_-+\tilde\phi_+)}(-\frac{1}{2}\tilde f\partial_+\tilde\psi_2 + \frac{1}{2}\tilde\psi_2\partial_+\tilde f + \partial_+\tilde\psi_1) \\
\tilde{\psi}_2'^\dagger &= -(k+\frac{1}{2})e^{\alpha_-(\tilde\phi_--\tilde\phi_+)}\partial_+\tilde\psi_2 - \frac{1}{2}\tilde\gamma\tilde{\psi}_1'^\dagger,
\end{aligned} \tag{22}$$

where we have defined

$$\alpha_- = -i\frac{\sqrt{2k+2}}{2k+1} = -i\frac{\alpha_+}{2k+1}, \tag{23}$$

and where normal ordering of the operators is implicit. We adopt the "conventional" normal ordering , which implies the following schematic rule for products of three or more operators,

$$\begin{aligned}
O_1O_2O_3 &= {}^{\times}_{\times}\, O_1 \;:\; O_2O_3 \;:\; {}^{\times}_{\times} \\
O_1O_2O_3O_4 &= \vdots\, O_1 \;{}^{\times}_{\times}\; O_2 \;: O_3O_4 \;:\; {}^{\times}_{\times} \vdots
\end{aligned} \tag{24}$$

The affine $\hat{sl}(2/1;\mathbf{R})$ Lie superalgebra is defined through the following operator product expansions between currents,

$$\begin{aligned}
J(\mathbf{e}_{\alpha_1+\alpha_2})(z)\ J(\mathbf{e}_{-(\alpha_1+\alpha_2)})(w) &\sim \frac{J(\mathbf{h}_-)(w)}{z-w} + \frac{k}{(z-w)^2} \\
J(\mathbf{h}_-)(z)\ J(\mathbf{e}_{\pm(\alpha_1+\alpha_2)})(w) &\sim \pm\frac{2J(\mathbf{e}_{\pm(\alpha_1+\alpha_2)})(w)}{z-w} \\
J(\mathbf{e}_{\pm(\alpha_1+\alpha_2)})(z)\ J(\mathbf{e}_{\mp\alpha_1})(w) &\sim \pm\frac{J(\mathbf{e}_{\pm\alpha_2})(w)}{z-w} \\
J(\mathbf{e}_{\pm(\alpha_1+\alpha_2)})(z)\ J(\mathbf{e}_{\mp\alpha_2})(w) &\sim \mp\frac{J(\mathbf{e}_{\pm\alpha_1})(w)}{z-w} \\
J(\mathbf{h}_-)(z)\ J(\mathbf{e}_{\pm\alpha_1})(w) \sim \pm\frac{J(\mathbf{e}_{\pm\alpha_1})(w)}{z-w} &, \qquad J(\mathbf{h}_-)(z)\ J(\mathbf{e}_{\pm\alpha_2})(w) \sim \pm\frac{J(\mathbf{e}_{\pm\alpha_2})(w)}{z-w} \\
J(\mathbf{h}_+)(z)\ J(\mathbf{e}_{\pm\alpha_1})(w) \sim \pm\frac{J(\mathbf{e}_{\pm\alpha_1})(w)}{z-w} &, \qquad J(\mathbf{h}_+)(z)\ J(\mathbf{e}_{\pm\alpha_2})(w) \sim \mp\frac{J(\mathbf{e}_{\pm\alpha_2})(w)}{z-w} \\
J(\mathbf{h}_\pm)(z)\ J(\mathbf{h}_\pm)(w) &\sim \mp\frac{k}{(z-w)^2} \\
J(\mathbf{e}_{\alpha_1})(z)\ J(\mathbf{e}_{-\alpha_1})(w) &\sim \frac{1}{2}\frac{(-J(\mathbf{h}_-)+J(\mathbf{h}_+)(w))}{z-w} - \frac{k}{(z-w)^2} \\
J(\mathbf{e}_{\alpha_2})(z)\ J(\mathbf{e}_{-\alpha_2})(w) &\sim \frac{1}{2}\frac{(J(\mathbf{h}_-)+J(\mathbf{h}_+))(w)}{z-w} + \frac{k}{(z-w)^2} \\
J(\mathbf{e}_{\pm\alpha_1})(z)\ J(\mathbf{e}_{\pm\alpha_2})(w) &\sim \frac{J(\mathbf{e}_{\pm(\alpha_1+\alpha_2)})(w)}{z-w}
\end{aligned} \tag{25}$$

As in the classical case, there exist two Wakimoto representations of this affine Lie superalgebra, which we will denote type I and type II since their classical limit (obtained when taking the level k to the ∞ limit) coincides with the type I and II representations (19),(20). The type I representation has been given by

[26] in their discussion of Hamiltonian reduction of the affine version of $Osp(2/2)$, and it appears as well in [24] and [27]. We shall include it here for completeness, and to allow direct comparison with the type II representation which we have not found in the literature. The type I free field representation is, with implicit conventional normal ordering,

$$
\begin{aligned}
J(\mathbf{h}_+) &= i\alpha_+\partial\phi_+ + {\psi_1'}^\dagger\psi_1' - {\psi_2'}^\dagger\psi_2' \\
J(\mathbf{h}_-) &= -i\alpha_+\partial\phi_- + {\psi_1'}^\dagger\psi_1' + {\psi_2'}^\dagger\psi_2' - 2\beta\gamma \\
J(\mathbf{e}_{\alpha_1+\alpha_2}) &= -i\alpha_+\gamma\partial\phi_- + \frac{1}{2}i\alpha_+\psi_1'\psi_2'\partial\phi_+ + \frac{1}{2}(k+1)(\partial\psi_1'\psi_2' + \partial\psi_2'\psi_1') \\
&\quad -k\partial\gamma + \gamma({\psi_1'}^\dagger\psi_1' + {\psi_2'}^\dagger\psi_2') - \beta\gamma^2 \\
J(\mathbf{e}_{-(\alpha_1+\alpha_2)}) &= \beta \\
J(\mathbf{e}_{\alpha_1}) &= -\frac{1}{2}i\alpha_+\psi_1'(\partial\phi_- + \partial\phi_+) - \frac{1}{2}(2k+1)\partial\psi_1' - \gamma{\psi_2'}^\dagger + \frac{1}{2}{\psi_2'}^\dagger\psi_2'\psi_1' - \frac{1}{2}\beta\gamma\psi_1' \\
J(\mathbf{e}_{-\alpha_1}) &= {\psi_1'}^\dagger + \frac{1}{2}\beta\psi_2' \\
J(\mathbf{e}_{\alpha_2}) &= -\frac{1}{2}i\alpha_+\psi_2'(\partial\phi_- - \partial\phi_+) - \frac{1}{2}(2k+1)\partial\psi_2' - \gamma{\psi_1'}^\dagger + \frac{1}{2}{\psi_1'}^\dagger\psi_1'\psi_2' - \frac{1}{2}\beta\gamma\psi_2' \\
J(\mathbf{e}_{-\alpha_2}) &= -{\psi_2'}^\dagger - \frac{1}{2}\beta\psi_1'.
\end{aligned}
\tag{26}
$$

For type II, one gets,

$$
\begin{aligned}
J(\mathbf{h}_+) &= i\alpha_+\partial\tilde\phi_+ + \tilde{\psi}_1'^\dagger\tilde\psi_1' + \tilde{\psi}_2'^\dagger\tilde\psi_2' \\
J(\mathbf{h}_-) &= -i\alpha_+\partial\tilde\phi_- + \tilde{\psi}_1'^\dagger\tilde\psi_1' - \tilde{\psi}_2'^\dagger\tilde\psi_2' - 2\tilde\beta\tilde\gamma \\
J(\mathbf{e}_{\alpha_1+\alpha_2}) &= -i\alpha_+\tilde\gamma\partial\tilde\phi_- - (k+\frac{1}{2})\partial\tilde\gamma + \frac{1}{2}\tilde\gamma(\tilde{\psi}_1'^\dagger\tilde\psi_1' - \tilde{\psi}_2'^\dagger\tilde\psi_2') - \tilde\beta\tilde\gamma^2 + \tilde\psi_1'\tilde{\psi}_1'^\dagger - \frac{1}{4}\tilde\gamma^2\tilde{\psi}_1'^\dagger\tilde\psi_2' \\
J(\mathbf{e}_{-(\alpha_1+\alpha_2)}) &= \tilde\beta - \frac{1}{2}\tilde{\psi}_1'^\dagger\tilde\psi_2' \\
J(\mathbf{e}_{\alpha_1}) &= -\frac{1}{2}i\alpha_+\tilde\psi_1'(\partial\tilde\phi_- + \partial\tilde\phi_+) + \frac{1}{4}i\alpha_+\tilde\gamma\tilde\psi_2'(3\partial\tilde\phi_- - \partial\tilde\phi_+) - \frac{1}{2}(k+1)\tilde\gamma\partial\tilde\psi_2' \\
&\quad +\frac{1}{2}(k-1)\partial\tilde\gamma\tilde\psi_2') - k\partial\tilde\psi_1' + \frac{1}{2}\tilde\beta\tilde\gamma^2\tilde\psi_2' - \tilde\beta\tilde\gamma\tilde\psi_1' + \tilde{\psi}_2'^\dagger\tilde\psi_1'\tilde\psi_2' \\
J(\mathbf{e}_{-\alpha_1}) &= \tilde{\psi}_1'^\dagger \\
J(\mathbf{e}_{\alpha_2}) &= \tilde{\psi}_2'^\dagger - \frac{1}{2}\tilde\gamma\tilde{\psi}_1'^\dagger \\
J(\mathbf{e}_{-\alpha_2}) &= \frac{1}{2}i\alpha_+\tilde\psi_2'(\partial\tilde\phi_- - \partial\tilde\phi_+) - \frac{1}{2}(2k+1)\partial\tilde\psi_2' - \tilde\beta\tilde{\psi}_1'^\dagger + \frac{1}{2}\tilde{\psi}_1'^\dagger\tilde\psi_2'\tilde\psi_1' + \frac{1}{2}\tilde\beta\tilde\gamma\tilde\psi_2',
\end{aligned}
\tag{27}
$$

where

$$\alpha_+ = \sqrt{2k+2}. \tag{28}$$

Having explicitly provided two genuinely different Wakimoto free field representations of $\hat{sl}(2/1;\mathbf{R})$, whose existence is intimately rooted in the non-uniqueness of the Dynkin diagram of $sl(2/1;\mathbf{R})$, it is useful to identify the relations between the free fields of these two representations. They read,

$$
\begin{aligned}
\gamma &= \tilde\gamma + \frac{1}{2}(\tilde\psi_1' - \frac{1}{2}\tilde\gamma\tilde\psi_2')e^{\alpha_-(\tilde\phi_- - \tilde\phi_+)}\tilde\psi_2 \\
\phi_- &= \tilde\phi_- - \frac{1}{2\alpha_-}e^{\alpha_-(\tilde\phi_- - \tilde\phi_+)}\tilde\psi_2'\tilde\psi_2 \quad , \quad \phi_+ = \tilde\phi_+ - \frac{1}{2\alpha_-}e^{\alpha_-(\tilde\phi_- - \tilde\phi_+)}\tilde\psi_2'\tilde\psi_2 \\
\psi_1' &= \tilde\psi_1' - \frac{1}{2}\tilde\gamma\tilde\psi_2' \quad , \quad \psi_2' = e^{\alpha_-(\tilde\phi_- - \tilde\phi_+)}\tilde\psi_2
\end{aligned}
$$

$$
\begin{aligned}
\beta &= \tilde{\beta} - \frac{1}{2}\tilde{\psi}_1'^{\dagger}\tilde{\psi}_2' \\
\psi_1'^{\dagger} &= \tilde{\psi}_1'^{\dagger} - \frac{1}{2}\tilde{\beta}e^{\alpha_-(\tilde{\phi}_- - \tilde{\phi}_+)}\tilde{\psi}_2 + \frac{1}{4}e^{\alpha_-(\tilde{\phi}_- - \tilde{\phi}_+)}\tilde{\psi}_1'^{\dagger}\tilde{\psi}_2'\tilde{\psi}_2 \\
\psi_2'^{\dagger} &= (k+\frac{1}{2})\partial_+\tilde{\psi}_2' - \frac{1}{2}i\alpha_+\tilde{\psi}_2'(\partial_+\tilde{\phi}_- - \partial_+\tilde{\phi}_+) + \frac{1}{2}\tilde{\beta}\tilde{\psi}_1' - \frac{1}{4}\tilde{\beta}\tilde{\gamma}\tilde{\psi}_2' - \frac{1}{4}\tilde{\psi}_1'^{\dagger}\tilde{\psi}_2'\tilde{\psi}_1'
\end{aligned}
\tag{29}
$$

It can be checked that substituting these relations into the type I currents (26) yields the type II expressions (27), provided the normal ordering defined in (24) is carefully implemented. In order to calculate the BRST cohomology on the whole Fock space of the $SL(2/1;\mathbf{R})/SL(2/1;\mathbf{R})$ topological model, one needs the Wakimoto representation (26) and its dual. The two are related by the following automorphism of order 4 of the $\hat{sl}(2/1;\mathbf{R})$ algebra,

$$
\begin{aligned}
J^{\pm} &\rightarrow -J^{\mp}, \qquad J^3 \rightarrow -J^3, \quad U \rightarrow -U \\
j^{\pm} &\rightarrow \pm j^{\mp}, \qquad j'^{\pm} \rightarrow \mp j'^{\mp}, \quad k \rightarrow -(k+2).
\end{aligned}
\tag{30}
$$

To conclude, we stress that in order to study the space of physical states of the noncritical $N=2$ theory, using the tool provided by topological G/G WZNW models, a detailed analysis of various modules over $A(1,0)^{(1)}$ is needed. Many Lie superalgebras share with $A(1,0)$ the property that two sets of simple roots may not be equivalent up to Weyl tranformations, which are generated by reflections with respect to bosonic simple roots. An added technical complication in $A(1,0)$ is the fact that the fermionic roots are lightlike, which prevents one from defining coroots and fundamental weights in a straightforward way. We have given the classical and quantum free field Wakimoto representations of $\hat{sl}(2/1;\mathbf{R})$ and shown that two Wakimoto representations built with two inequivalent sets of simple roots are different. Classically, the relation between the two can be derived from first principles. Remarkably, there also exists a set of field transformations which relate the two Wakimoto representations in the quantum case.

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